Solving the Enneomino Puzzle in CFF 85

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In the given problem, we started with a square with 9 little squares located as far away from each other as possible. In Figure 1 you can see a tenth square that has been added adjacent to the upper left one, which can be chosen due to diagonal symmetry. Next, because piece 1 is not allowed to contain a 2x2-square, one of the squares adjacent to the present piece 1 must be part of piece 2. These options are also shown in Figure 1.

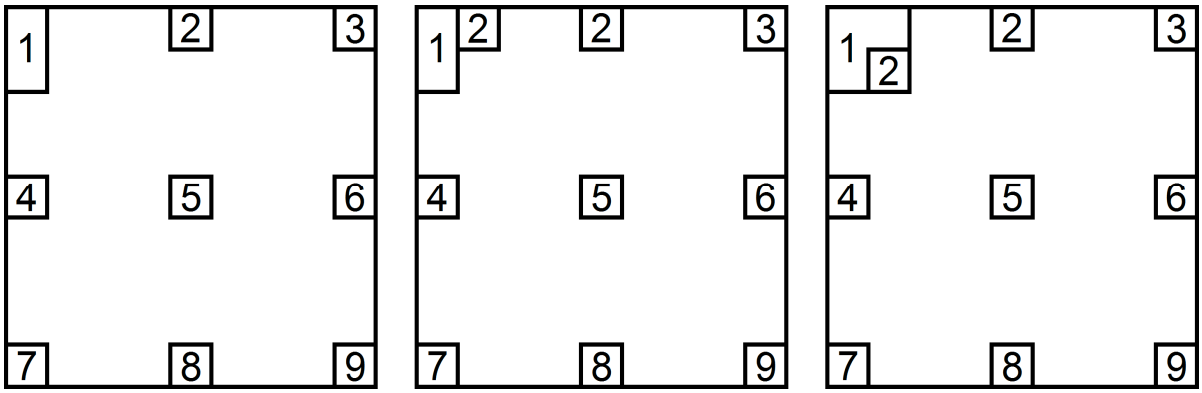

Figure 1. Left: a tenth square added
Middle and right: two possible choices

In both situations we can draw the same rectangle within which both pieces 1 and 2 must be confined, because all pieces must fit within a 4x4-square. This is shown in the left image of Figure 2. Then, next to piece 2 on the right, another rectangle can be drawn in which pieces 3 and 6 must be placed, shown in the middle of figure 2. In the right image in Figure 2, all four rectangles are drawn that can be placed with similar logic.

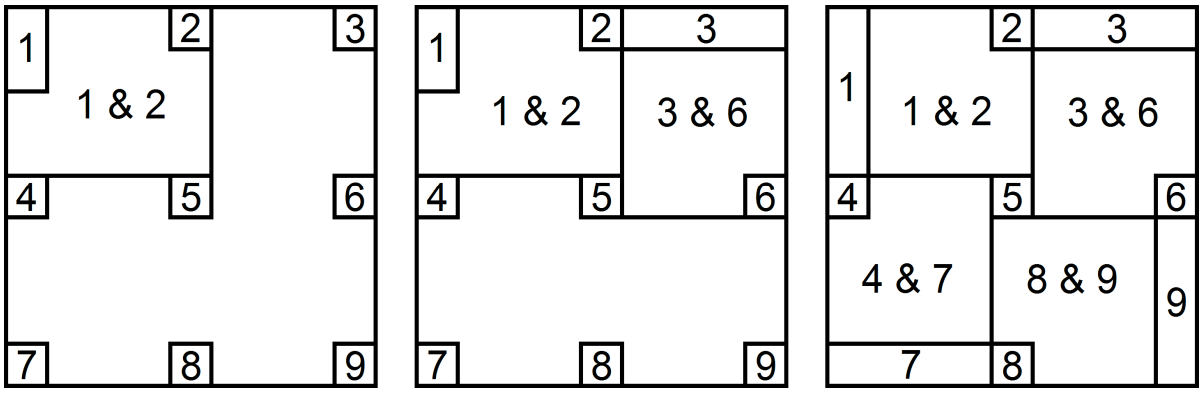

Figure 2. Continuing logic

In the left part of Figure 3 a square has been added within which piece 5 must be placed. Considering symmetry, only one location for this square remains, as within each 4x5-rectangle there must be two squares filled by piece 5, as two enneominoes cover just a size 18 area within the size 20 area of these rectangles. The immediate consequences of drawing this square are shown in the middle of Figure3, followed by yet another consequence for pieces 4 and 5, as piece 5 may not contain a 2x2-square.

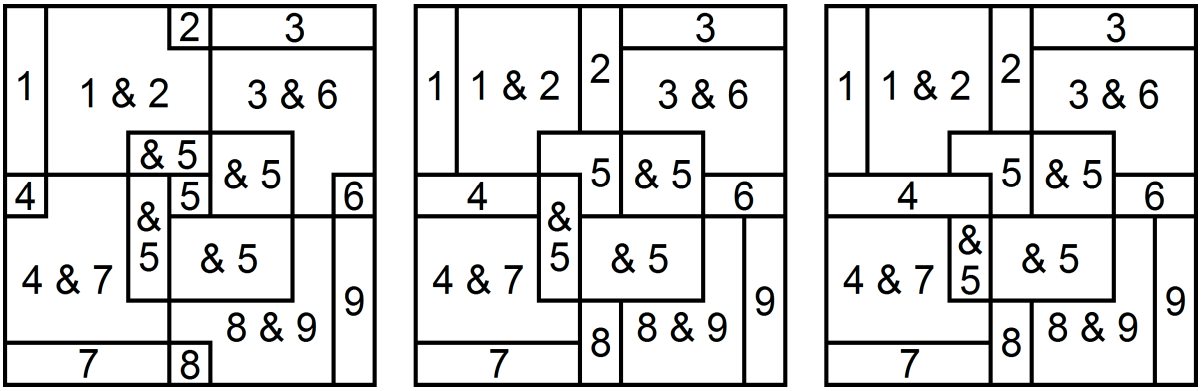

Figure 3. Continuing logic

There are just two choices for the area covered by piece 5 within the rectangle for pieces 3 and 6, because piece 5 may not contain a 2x2-square and piece 5 may not cover more than two squares in the rectangle for pieces 8 and 9. Also, the two remaining squares of piece 5 are fixed, that occur within the rectangle for pieces 4 and 7. This is shown in Figure 4.

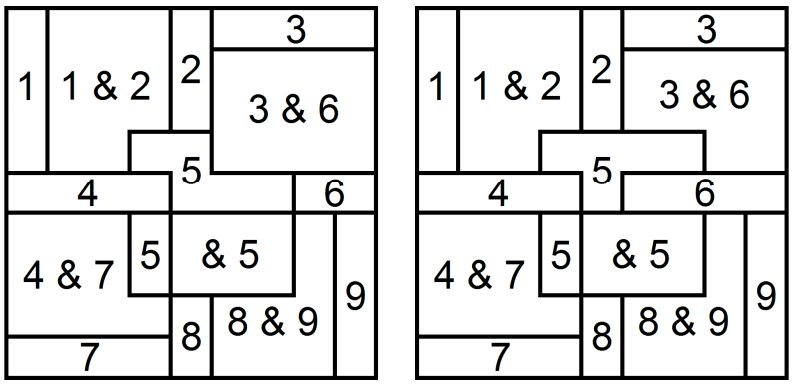

Figure 4. Continuing logic, with two different options

For both choices, there is just one way to complete the shape of piece 5 as well as there is just one way to place two slim enneominoes within the remaining part of the rectangle for pieces 4 and 7. The remains of the rectangle for pieces 3 and 6 in the left choice has two options, whereas for the right choice there is just one option. These possibilities for the left choice are shown in the left and central images of Figure 5. The continuing logic of the situation in the right image of Figure 4 is shown in Figure 6.

In the right image of Figure 5, the only way to fill the remaining part of the rectangle for pieces 8 and 9 in the left and central images of Figure 5 with two slim

enneominoes is shown. As piece 7 and piece 9 are equal in shape, this possibility is rejected.

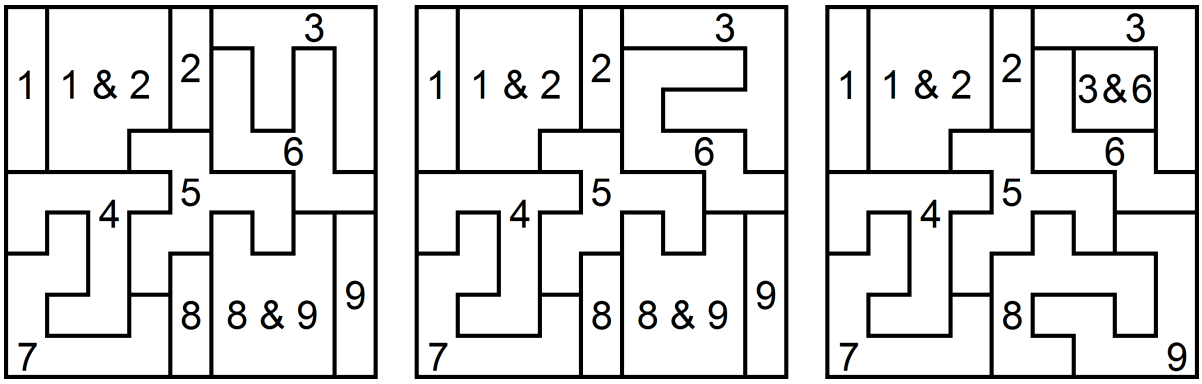

Figure 5. The continuing logic of the left image of Figure 4 leads to a dead end, in the right image pieces 7 and 9 are equal in shape

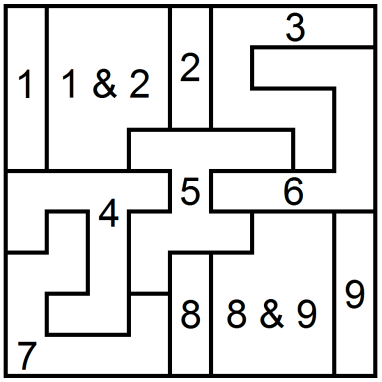

Figure 6. Continuing logic of the right image of Figure 4

After the elimination of the images in Figure 5, the only possibility left over is the situation shown in Figure 6. The remaining parts of the rectangles for pieces 1, 2, 8 and 9 are left to be filled in. As these remaining parts are equal in shape, luckily this time there are two possibilities to fill such a shape with two slim enneominoes, leaving two options to place them. Both options are drawn in Figure 7.
It now has been proven that there is only one set of 9 different slim enneominoes each fitting inside a 4x4-square suitable for filling a 9x9-square.

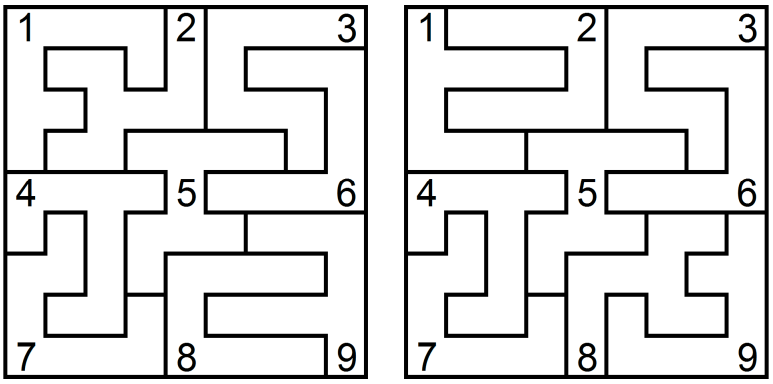

Figure 7. The final solution, with both options